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REPRESENTATION OF MULTIPHASE WELLBORE EFFECTS IN PRESSURE TRANSIENT ANALYSIS

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In the analysis of pressure transient tests in wells, classical methods have approximated flow within the reservoir and the wellbore using single-phase flow equations. The advantage to these equations is that they are well known and fairly easily solved. In addition, it has been found that these equations seem to represent the transient well performance fairly well. Unfortunately, the actual situation is more complicated, with multi-phase flow effects (such as phase redistribution in the wellbore and fluid fronts in the reservoir) distorting the pressure response predicted by single-phase equations. In addition, the standard equations do not directly allow for the influx of external mass or the independent control of surface flow for several flowing streams. In actual field operations, external mass influx may represent gas injected into the wellbore for gas lift or oil injected for hydraulic pump operation. It is also common practice to produce oil from the tubing and gas up the casing annulus in beam pumped wells, whereby the flowing streams are controlled independently at the surface.

In this note, the effect of multi-phase flow in the wellbore is evaluated using macroscopic mass and momentum balances. It is shown that multi-phase flow in the wellbore can be represented in a manner similar to the normal equations for wellbore storage, if the definition of the parameters is modified to account for all of the mass flowing within the well. Representation of multi-phase flow in the reservoir is beyond the scope of this note.

The basic transport equations for fluid flow in the wellbore consist of the conservation of mass and the conservation of linear momentum[1]. In general, we will consider one mass balance equation for each flowing component and one momentum balance over the wellbore. Following the approach of Fair[2] (1992) and assuming that momentum effects and friction are negligible, the momentum balance equation reduces to a force balance. The mass balance relations simply state that the change in mass of each component must equal the net mass influx of the component. These relations are shown in 1, for a fluid system consisting of oil and gas components. Note that we allow the addition of external mass, so that gas lift and other wellbore processes can be represented.

$$\left(q_o\rho_o\right)_s-\left(q_o\rho_o\right)_w+\left(q_o\rho_o\right)_e=\frac{dm_o}{dt}$$

$$\left(q_g\rho_g\right)_s-\left(q_g\rho_g\right)_w+\left(q_g\rho_g\right)_e=\frac{dm_g}{dt} \quad \text{.......(1)}$$

$$p_w-p_s=\frac{g}{Ag_c}\left(m_o+m_g\right)$$

By summing the mass balance equations and differentiating the force balance, Equation 2 is obtained. It should be noted that these equations could be generalized for any number of components. For each component, a mass balance relation would be written, and then all of the relations would be summed. It can be shown that by summing the mass balance equations, inter-phase mass transfer is also automatically accounted for since the equation represents total mass within the wellbore.

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_s-\left[(q_o\rho_o)+(q_g\rho_g)\right]_w+\left[(q_o\rho_o)+(q_g\rho_g)\right]_e=\frac{dm_t}{dt}$$

$$\frac{dp_w}{dt}-\frac{dp_s}{dt}=\frac{g}{Ag_c}\frac{dm_t}{dt} \qquad \text{(2)}$$

$$m_t=m_o+m_g$$

Following the procedure of Fair[2] (1992), the mass derivative can be expanded in terms of partial derivatives with respect to pressure and time. This is shown in Equation 3. Substituting this into the balance equations yields 4.

$$\frac{dm_t}{dt}=\frac{\partial m_t}{\partial p_w}\left(\frac{dp_w}{dt}+\frac{\partial m_t/\partial t}{\partial m_t/\partial p_w}\right)=\frac{\partial m_t}{\partial p_w}\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_{m_t}\right) \qquad \text{(3)}$$

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_s-\left[(q_o\rho_o)+(q_g\rho_g)\right]_w+\left[(q_o\rho_o)+(q_g\rho_g)\right]_e$$
$$=\frac{\partial m_t}{\partial p_w}\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right)$$

$$\text{(4)}$$

$$\frac{dp_w}{dt}-\frac{dp_s}{dt}=\frac{g}{Ag_c}\frac{\partial m_t}{\partial p_w}\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right)$$

Rearranging the force balance, the surface pressure relation becomes Equation 5. This is completely analogous to the relation derived in a previous note[3] for single-phase flow.

$$\frac{dp_s}{dt}=\left(1-\frac{g}{Ag_c}\frac{\partial m_t}{\partial p_w}\right)\frac{dp_w}{dt}+\left(\frac{g}{Ag_c}\frac{\partial m_t}{\partial p_w}\right)\left.\frac{dp_w}{dt}\right|_m \qquad \text{(5)}$$

The mass balance relation is repeated in Equation 6 This equation has the same general form as the normal equation for wellbore storage with phase redistribution[2], except for the external mass term on the left side of the equation.

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_s-\left[(q_o\rho_o)+(q_g\rho_g)\right]_w+\left[(q_o\rho_o)+(q_g\rho_g)\right]_e$$
$$=\frac{\partial m_t}{\partial p_w}\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right) \qquad \text{(6)}$$

These equations can be written in a somewhat simplified form by defining the storage and density parameters as follows.

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_s-\left[(q_o\rho_o)+(q_g\rho_g)\right]_w+\left[(q_o\rho_o)+(q_g\rho_g)\right]_e = C\rho_w\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right)$$

$$\frac{dp_s}{dt}=\left(1-\frac{C}{C_\rho}\right)\frac{dp_w}{dt}+\left(\frac{C}{C_\rho}\right)\left.\frac{dp_w}{dt}\right|_m \qquad (7)$$

$$C=\frac{1}{\rho_w}\frac{\partial m_t}{\partial p_w}$$

$$C_\rho=\frac{Ag_c}{\rho_w g}$$

It would appear at this point that if the external mass flux is ignored, we could simply define the flow rates to be total flow rates and the storage related parameters to be based on total mass and the standard equations could be used. There is, however, a somewhat subtle difference. To apply these wellbore equations, it will generally be necessary to couple the component flow rates at the reservoir through a diffusivity equation for each phase and PVT relationships. At the surface, however, the relationship between the flow rates may or may not be coupled, depending on how the well is operated. In a flowing or gas lift well, the surface rates are most likely coupled, since they are generally controlled by a single valve or choke. In a beam or submersible pumped well, however, the gas may flow up the annulus while the oil flows up the tubing, so the surface controls for the component flow rates are independent.

If we assume that the gas mass flux at the bottom of the well is proportional to the oil mass flux, then we can write the bottomhole flux term as Equation 8. (R is the producing gas-oil mass ratio. Note that the normal gas-oil ratio is a volume ratio at standard conditions. Note also that R is not necessarily constant.)

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_w=(q_o\rho_o)_w(1+R)=q_{ow}\rho_{ow}(1+R)$$

$$R=\frac{(q_g\rho_g)_w}{(q_o\rho_o)_w} \qquad (8)$$

If we define the bottomhole density to include the gas density, then the mass balance equation can be written as Equation 9.

$$\left[(q_o\rho_o)+(q_g\rho_g)\right]_s-q_{ow}\rho_{wo}(1+R)+\left[(q_o\rho_o)+(q_g\rho_g)\right]_e = C\rho_{wo}(1+R)\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right) \qquad (9)$$

Dividing by the bottomhole density puts the equation into a form for wellbore storage and phase redistribution, as shown in Equation 10.

$$q_{os}\left[\frac{\rho_{os}}{\rho_{wo}(1+R)}\right]+q_{gs}\left[\frac{\rho_{gs}}{\rho_{wo}(1+R)}\right]-q_{ow}+q_{oe}\left[\frac{\rho_{oe}}{\rho_{wo}(1+R)}\right]+q_{ge}\left[\frac{\rho_{ge}}{\rho_{wo}(1+R)}\right] = C\left(\frac{dp_w}{dt}-\left.\frac{dp_w}{dt}\right|_m\right) \qquad (10)$$

In this form, note that the density ratios are equivalent to formation volume factors, but must include the effect of the gas, as well as the effect of the oil. Rearranging this equation and defining the appropriate volume factors yields Equation 11.

$$q_{ow} = \left(q_{os}B_o + q_{gs}B_g\right) + \left(q_{oe}B_{oe} + q_{ge}B_{ge}\right) - C\left(\frac{dp_w}{dt} - \left.\frac{dp_w}{dt}\right|_m\right) \qquad (11)$$

It should also be noted that from the force balance equation, the constant mass pressure derivative is the same whether it is based on surface or bottomhole pressure. Thus, we are free to compute it however is most convenient.

$$\left.\frac{dp_w}{dt}\right|_m = \left.\frac{dp_s}{dt}\right|_m \qquad (12)$$

In summary, we have derived a general wellbore model that describes multi-phase flow in the wellbore based on the macroscopic mass and momentum balance relations. We have assumed that momentum and friction are negligible in the wellbore, as well as assume that there is a relationship between the oil and gas flow rates at the bottom of the well. (It is fairly straightforward to remove the momentum assumption by following the procedures in Reference 1.) The resulting equations resemble the form of the single-phase equations quite closely, which explains why the single-phase equations have been so successfully used for many years. We note, however, that the definitions of the wellbore storage and density parameters are somewhat different, in that they must account for the mass of all the components. Note that we have not assumed that any of the parameters are constant, although they may be approximately constant in practice.

In addition, the model derived here allows for the mass flux due to an external stream. In the case of a gas lifted well, this can represent the lift gas, while in a hydraulically pumped well, this might be the hydraulic oil stream. We also note that in artificially lifted wells, the standard well testing boundary conditions might be more complex, since there is the opportunity for multiple, independent flow rate controls. In a gas lift well, we can open, close, or choke the surface production, but the lift gas can be controlled separately. Similarly, in a beam pumped well, the pump can be on or off, controlling the surface oil flow, but the casing valve can also be open, closed, or choked, controlling the gas flow rate independently. It appears possible that these observations might allow for modified well testing methods suitable for specific applications, especially in the areas of well performance evaluation and artificial lift problem diagnosis.

Nomenclature

A cross-sectional area of wellbore

B formation volume factor

g, g_c gravitational constant

m mass

p pressure

q volumetric flow rate

R gas-oil mass ratio

t time

ρ density

Subscripts

e external

g gas

o oil

s surface

t total

w wellbore

References

1. Winterfeld, P. H., "Simulation of Pressure Buildup in a Multiphase Wellbore Reservoir System," *SPE Formation Evaluation*, June 1989.

2. Fair, Walter B. Jr., "Generalization of Wellbore Effects in Pressure Transient Analysis," SPE 24715, Society of Petroleum Engineers, Washington, DC, 1992.

3. Fair, W. B. Jr., "The Computation of Surface Pressures in Pressure Transient Analysis," TechNotes, The ComPort Computing Co., Houston, TX, June 1993.