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THE COMPUTATION OF SURFACE PRESSURES IN PRESSURE TRANSIENT ANALYSIS

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In pressure transient analysis, pressures are usually measured at the bottom of the well, however, there are occasions when that is impossible or expensive. In other cases, it would be desirable to use surface pressure measurements to determine well performance and make preliminary evaluations of well and reservoir behavior. In nearly all cases, measuring surface pressures is relatively easy, inexpensive, and minimizes the mechanical risk involved in down-hole gage placement and retrieval.

In this note, an expression for the surface pressure in a well in terms of the wellbore storage, constant mass pressure change and the bottomhole pressure is derived. It is shown that to compute the surface pressure, an additional parameter is needed. That parameter represents the relative fluid density in the wellbore. It is also shown that in many cases the surface pressures will not reflect the bottomhole conditions precisely, implying that in many cases the direct analysis of surface pressure data will be difficult or impossible.

To derive the equation for surface pressure, we start with the fundamental equations for fluid transport in the wellbore. Using macroscopic mass balance and momentum balance relations as documented by Fair (1992),[1] and assuming that momentum and friction are negligible, Equations 1 and 2 result. Note that the momentum balance degenerates to a force balance for these assumptions.

$$(q\rho)_s - (q\rho)_w = \frac{dm}{dt} = C\rho_w\left(\frac{dp_w}{dt} - \frac{dp_\phi}{dt}\right) \quad (1)$$

$$p_w - p_s = \frac{mg}{Ag_c} \quad (2)$$

Differentiating Equation 2 with respect to time, substituting the mass derivative from Equation 1, and rearranging yields Equation 3, which describes the surface pressure change for a well test.

$$\frac{dp_s}{dt} = \left(1 - \frac{C}{C_\rho}\right)\frac{dp_w}{dt} + \frac{C}{C_\rho}\frac{dp_\phi}{dt}$$
$$C_\rho = \frac{Ag_c}{\rho_w g}$$
$$C = \frac{1}{\rho_w}\frac{\partial m}{\partial p_w} \qquad \text{(3)}$$
$$\frac{dp_\phi}{dt} = \frac{\partial m / \partial t}{\partial m / \partial p_w} = \left(\frac{dp_w}{dt}\right)_m$$

In Equation 3, C represents the usual wellbore storage parameter and C_ρ represents the storage due only to the fluid density. In the case of a rising fluid level, C_ρ is the effect of the fluid level rise, while C is the combined effect of both fluid level rise and fluid compressibility. This is shown in Equation 4.

$$C = \frac{1}{\rho_w}\frac{\partial m}{\partial p_w} = \frac{\bar{\rho}}{\rho_w}\frac{\partial V}{\partial p_w} + \frac{V}{\rho_w}\frac{\partial \bar{\rho}}{\partial p_w} = A\frac{\bar{\rho}}{\rho_w}\frac{\partial x_L}{\partial p_w} + \frac{Ax_L\bar{\rho}}{\rho_w}\frac{\partial \bar{\rho}}{\partial p_w} \qquad \text{(4)}$$

Note that if $C = C_\rho$, the surface pressure is not affected by the bottomhole pressure and is a reflection of the constant mass pressure change. In other words, if the surface pressure changes, it is not due to a change in the bottomhole pressure, which may or may not change. This can happen when the wellbore fluid is incompressible, so the wellbore mass cannot change, since p_ϕ is equal to the pressure change at constant wellbore mass. It can also occur when there is a rising liquid level with negligible gas compressibility above the liquid. In that case, all of the bottomhole pressure change is explained by the changing column height and the surface pressure does not change due to the bottomhole pressure or the change in wellbore mass.

At the other extreme, if the constant mass pressure change is zero, then the surface pressure will reflect the bottomhole pressure. Note that in this case the surface pressure will change in proportion to the bottomhole pressure, with the ratio determined by the ratio of the wellbore storage to the fluid density. This ratio is a relative measure of the wellbore fluid compressibility.

In applying the surface pressure relation derived above, there are generally 3 cases of special interest: 1) wellbore is full of a slightly compressible fluid (oil), 2) wellbore is full of a very compressible fluid (gas), and 3) wellbore has a changing fluid level. The implications of these situations are further described below.

Wellbore full, slightly compressible fluid (oil)

If the wellbore is full of a slightly compressible fluid, then the storage parameter, C, becomes Equation 5, while the ratio C/C_ρ becomes Equation 6.

$$C = \frac{V\bar{\rho}}{\rho_w}\frac{1}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial p_w} = V\left(\frac{\bar{\rho}}{\rho_w}\right)c_f \qquad \text{(5)}$$

$$\frac{C}{C_\rho} = V\left(\frac{\bar{\rho}g}{g_c}\right)c_f \qquad \text{(6)}$$

In this case, the fluid compressibility, c_f, will be small so that C/C_ρ is also small. In addition, since we have assumed a single phase, the constant mass pressure change would most likely be small, unless there are significant temperature effects. This implies that the surface pressure equation will be sensitive to the bottomhole pressure. For this situation, pressure transient analysis based on surface pressure measurements should be feasible.

Wellbore full, very compressible fluid (gas)

If the wellbore is full of a very compressible fluid, such as gas, the relations of Equations 5 and 6 still apply. In this case, however, the compressibility will not necessarily be constant and the situation would appear to be more complex. If, however, we use the real gas equation of state to express the density and fluid compressibility, the ratio of Equation 6 can be expressed in Equation 7.

$$\frac{C}{C_\rho} = V\left(\frac{\bar{\rho}g}{g_c}\right)\left(\frac{1}{\bar{p}} - \frac{1}{z}\frac{\partial z}{\partial \bar{p}}\right) \quad (7)$$

Since the z-factor derivative is generally small at low pressures and the gas density is approximately proportional to pressure, the ratio will be approximately constant. (Note that at high pressures, above about 1000 psi, this is not true.) Whether it is approximately constant or not, the ratio will be much higher than for the slightly compressible fluid case. Since we have assumed a single phase, the constant mass pressure change would most likely be small, except for temperature effects. Therefore, it would be expected that pressure transient analysis based on surface pressure may be feasible for gas wells, but with reduced sensitivity to reservoir parameters, since the pressure change at the surface will be less than the pressure change at the bottom of the wellbore. Note, however, that in the limit of low pressure, the density becomes small and at high pressures the compressibility becomes small. In these pressure ranges, enough sensitivity to bottomhole pressure changes should be observed.

Changing liquid level

For a changing liquid level, the situation is more complex, with Equation 4 having both terms. We can approximate the situation by assuming the liquid is incompressible and the gas above is weightless and ideal. In this case, the second term in Equation 4 can be neglected. If we consider 1 unit length of fluid rise, the volume of fluid would equal A, the wellbore cross-sectional area. The pressure would rise by an amount equivalent to the fluid density, ρ_w, plus an additional amount due to the compression of the gas above the fluid level, $\Delta p_s = p_s (D - x_L)/(D - x_L - 1)$. This allows us to estimate the storage C by Equation 8 and the ratio in Equation 9.

$$C \approx \frac{A}{\rho_w + \left(\frac{p_s}{D - x_L - 1}\right)} \quad (8)$$

$$\frac{C}{C_\rho} \approx \frac{\rho_w\left(g/g_c\right)}{\rho_w + \left(\frac{p_s}{D - x_L}\right)} \quad (9)$$

From Equation 9, we can see that as the surface pressure decreases and the amount of liquid in the well ($D - x_L$) decreases (the fluid level from the surface increases), that the ratio becomes close to 1. Under those circumstances, conducting well tests based on surface pressures would be very indeterminate, since the surface pressure change would have little relationship to the bottomhole pressure change. However, if the surface pressure is relatively high and the fluid level is closer to the surface ($D - x_L$ is small), the surface pressure is more sensitive, but the parameters become more non-linear. Under these circumstances, the possibility for conducting well tests based on surface pressure data might be feasible, but the interpretation may have to properly account for the non-linearity in the resulting equations.

Nomenclature

A	cross-sectional area of wellbore
c_f	fluid compressibility
C	wellbore storage parameter
C_ρ	wellbore fluid density parameter

D	bottomhole depth of wellbore
g, g_c	gravitational constant
m	mass
p_s	surface pressure
p_w	bottomhole pressure
q	volumetric flow rate
t	time
V	wellbore fluid volume
x_L	height of fluid column in wellbore
ρ	density

References

1. Fair, Walter Jr., “Generalization of Wellbore Effects in Pressure Transient Analysis,” SPE 24715, Society of Petroleum Engineers, Washington, DC, 1992.